PERSONAL DATA

First Name:	MICHELE
Surname:	RIMOLDI
Born:	CANTÙ (CO), 17/11/1984
Nationality:	ITALIAN
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CURRENT POSITION

Since January 2017	Researcher (RTD-a) Politecnico di Torino Dipartimento di Scienze Matematiche "Giuseppe Luigi Lagrange"

PREVIOUS POSITIONS

Oct. 2016-Dec. 2016	"Emma e Giovanni Sansone" Junior visiting position Centro di Ricerca Matematica "Ennio De Giorgi" Scuola Normale Superiore (Pisa, Italy)
Dec. 2015-Sept. 2016	Post-Doc position (1 year) Funded by the Fondation Sciences Mathématiques de Paris (FSMP) Laboratoire Analyse Géométrie et Applications (LAGA) Institut Galilée Université Paris 13 - Sorbonne Paris Citè
Jan. 2014 - Nov. 2015	Assegno di Ricerca (2 years Post-Doc position), Università degli Studi di Milano-Bicocca Dipartimento di Matematica e Applicazioni
Dec. 2011 - Dec. 2013	Assegno di Ricerca (2 years Post-Doc position), under the supervision of Prof. Stefano Pigola, Università degli Studi dell'Insubria (Como, Italy) Dipartimento di Scienza e Alta Tecnologia

ACADEMIC HABILITATIONS

December 2014	National Italian habilitation as *Professore di II fascia* in Geometry and Algebra. Sector: 01/A2. Validity: 30/12/2014–30/12/2020

February 2016	Qualification Française aux fonctions de *Maître de conférences.* 25ème section.

EDUCATION

March 2012	Ph.D. in Mathematics, Università degli Studi di Milano, Italy Thesis–title: *Rigidity results for Lichnerowicz Bakry–Émery Ricci tensors* Supervisor: Prof. Stefano Pigola
September 2008	Laurea specialistica (master degree) cum laude in Mathematics Università degli Studi dell'Insubria (Como, Italy) Thesis–title: *Caratterizzazione di modelli mediante equazioni differenziali* Supervisor: Prof. Stefano Pigola
January 2007	Laurea triennale (bachelor) cum laude in Mathematics Università degli Studi dell'Insubria (Como, Italy) Thesis–title: *Costruzione di superfici minime complete tra due piani paralleli in* $\mathbb{R}^3$ Supervisor: Prof. Stefano Pigola
July 2003	Diploma di Maturità Liceo Scientifico Galileo Galilei (Erba (CO), Italy)

PUBLICATIONS

[MMR] C. Mantegazza, S. Mongodi, M. Rimoldi
The Cotton tensor and the Ricci flow
To appear on Geom. Flows.
(Preliminary version on **arXiv:1203.4433v3**)

[IR2] D. Impera, M. Rimoldi
Rigidity results and topology at infinity of translating solitons of the mean curvature flow
Communications in Contemporary Mathematics.
19, 1750002 (2017), no. 6, [21 pages], doi: 10.1142/S021919971750002X

[IR1] D. Impera, M. Rimoldi
Stability properties and topology at infinity of f-minimal hypersurfaces
Geometriae Dedicata
178 (2015), no. 1, 21-47, doi: 10.1007/s10711-014-9999-6

[RV1] M. Rimoldi, G. Veronelli
Topology of steady and expanding gradient Ricci solitons via f-harmonic maps
Differential Geometry and its Applications
31 (2013), no. 5, 623-638, doi: 10.1016/j.difgeo.2013.06.001.

[PR2] S. Pigola, M. Rimoldi
Complete self-shrinkers confined into some regions of the space
Annals of Global Analysis and Geometry
45 (2014), no. 1, 47-65, doi: 10.1007/s10455-013-9387-8

[MR] P. MASTROLIA, M. RIMOLDI
Some triviality results for quasi–Einstein manifolds and Einstein warped products
GEOMETRIAE DEDICATA
169 (2014), no. 1, 225-237, doi: 10.1007/s10711-013-9852-3

[R2] M. RIMOLDI
On a classification theorem for self-shrinkers
PROCEEDINGS OF THE AMERICAN MATHEMATICAL SOCIETY
142 (2014), no. 10, 3605-3613, doi: 10.1090/S0002-9939-2014-12074-0

[MRR] P. MASTROLIA, M. RIGOLI, M. RIMOLDI
Some geometric analysis on generic Ricci solitons
COMMUNICATIONS IN CONTEMPORARY MATHEMATICS
15 (2013), no. 3, 1250058, 25 pp., doi: 10.1142/S0219199712500587.

[CMMR] G. CATINO, C. MANTEGAZZA, L. MAZZIERI, M. RIMOLDI
Locally conformally flat quasi–Einstein manifolds
JOURNAL FÜR DIE REINE UND ANGEWANDTE MATHEMATIK
675 (2013), 181–189, doi: 10.1515/crelle.2011.183.

[R1] M. RIMOLDI
A remark on Einstein warped products
PACIFIC JOURNAL OF MATHEMATICS
252 (2011), no. 1, 207–218, doi: 10.2140/pjm.2011.252.207

[MRV] P. MASTROLIA, M. RIMOLDI, G. VERONELLI
Myers–type theorems and some related oscillation results
JOURNAL OF GEOMETRIC ANALYSIS
22 (2012), no. 3, 763–779, doi:10.1007/s12220-011-9213-0

[PRRS] S. PIGOLA, M. RIGOLI, M. RIMOLDI, A. G. SETTI
Ricci almost solitons
ANNALI DELLA SCUOLA NORMALE SUPERIORE DI PISA. CLASSE DI SCIENZE
X (2011), no. 4, 757–799, doi: 10.2422/2036-2145.2011.4.01

[PRiS] S. PIGOLA, M. RIMOLDI, A. G. SETTI
Remarks on non–compact gradient Ricci solitons
MATHEMATISCHE ZEITSCHRIFT
268 (2011), no. 3–4, 777–790, doi: 10.1007/s00209-010-0695-4

[PR1] S. PIGOLA, M. RIMOLDI
Characterizations of model manifolds by means of certain differential systems
CANADIAN MATHEMATICAL BULLETIN
55 (2012), 632–645, doi:10.4153/CMB-2011-134-0

PREPRINTS

[RV2] M. RIMOLDI, G. VERONELLI
Extremals of Log Sobolev inequality on non-compact manifolds and Ricci soliton structures
(submitted, **arXiv:1605.09240v2**)

TALKS, SEMINARS AND POSTERS

April 2017 — UNIVERSITÀ DI TORINO, TORINO, ITALY
Differential Geometry Days
Invited talk:
"Extremals of Log Sobolev inequalities on non-compact manifolds and Ricci soliton structures"

February 2017 — SCUOLA NORMALE SUPERIORE, PISA, ITALY
Workshop su varietà reali e complesse geometria, topologia e analisi armonica
Invited talk:
"Extremals of Log Sobolev inequalities on non-compact manifolds and Ricci soliton structures"

December 2016 — SCUOLA NORMALE SUPERIORE, PISA, ITALY
Seminari di Analisi Geometrica
Seminar:
"Extremals of Log Sobolev inequalities on non-compact manifolds and Ricci soliton structures"

September 2016 — UNIVERSITÀ DI CAGLIARI, CAGLIARI, ITALY
Complex and Riemannian Geometry days
Invited talk:
"Extremals of Log Sobolev inequalities on non-compact manifolds and Ricci soliton structures"

June 2016 — UNIVERSITÉ PARIS 13, PARIS, FRANCE
Groupe de travail Analyse semi-classique
Seminar:
"Stability properties and topology at infinity of f-minimal hypersurfaces"

April 2016 — UNIVERSITÉ PARIS 7, PARIS, FRANCE
Séminaire de Géométrie
Invited talk:
"Properness and boundedness properties of complete self-shrinkers for the mean curvature flow"

Mars 2016 — UNIVERSITÉ PARIS 13, PARIS, FRANCE
Séminaire EDP non linéaires
Invited talk:
"Obstructions on the topology at infinity of translating solitons of the mean curvature flow"

April 2015 — UNIVERSIDAD DE GRANADA, GRANADA, SPAIN
Workshop on Geometric Flows
Invited talk:
"Rigidity results and topology at infinity of translating solitons of the mean curvature flow"

February 2015 — UNIVERSITÀ DI CAGLIARI, CAGLIARI, ITALY
Seminari di Matematica
Seminar:
"Complete self–shrinkers confined into some regions of the space"

November 2014	UNIVERSITÀ DEGLI STUDI DI MILANO, MILANO, ITALY *PDE's and Global Analysis @ UniMi* Invited talk: *"Complete self–shrinkers confined into some regions of the space"*
October 2014	LEVICO TERME, TRENTO, ITALY *Progressi Recenti in Geometria Reale e Complessa - IX* Invited talk: *"Complete self–shrinkers confined into some regions of the space"*
February 2014	UNIVERSIDADE FEDERAL DO CEARÀ, FORTALEZA, BRASIL *VII Workshop on Geometric Analysis* Invited talk: *"Complete self–shrinkers confined into some regions of the space"*
April 2011	CENTRO DI RICERCA MATEMATICA ENNIO DE GIORGI, PISA, ITALY *Ricci Solitons Days in Pisa 2011* Contributed talk: *"Triviality results for quasi–Einstein metrics and Einstein warped products"*
February 2011	UNIVERSIDAD DE GRANADA, SPAIN *Spanish–Japanese Workshop on Differential Geometry in Granada* Poster: *"Triviality results for quasi–Einstein metrics and Einstein warped products"*
January 2011	UNIVERSITÀ DEGLI STUDI DELL'INSUBRIA, ITALY *Seminari di Analisi Geometrica* Seminar: *"Triviality results for quasi–Einstein metrics and Einstein warped products"*
January 2011	UNIVERSITÀ DEGLI STUDI DELL'INSUBRIA, ITALY *Seminari di Analisi Geometrica* Seminar: *"Quasi–Einstein metrics and Einstein warped products"*

INVITATIONS

February 2015	UNIVERSITÀ DI CAGLIARI, CAGLIARI, ITALY (1 WEEK)
February 2014	UNIVERSIDADE FEDERAL DO CEARÀ, FORTALEZA, BRASIL (2 WEEKS)
Sept.-Dec. 2012	INSTITUT HENRI POINCARÉ, PARIS, FRANCE (3 MONTHS) Guest for the research trimester in *Conformal and Kälher Geometry*
January 2011	SCUOLA NORMALE SUPERIORE, PISA, ITALY (1 WEEK)

REFEREE ACTIVITY

Referee for: Proceedings of the Edinburgh Mathematical Society, Communications in Analysis and Geometry, Journal of Geometric Analysis, Journal of Mathematical Analysis and Applications, Illinois Journal of Mathematics, International Journal of Mathematics, Bulletin des Sciences Mathematiques, Annali di Matematica Pura ed Applicata, Differential Geometry and its Applications, International Journal of Geometric Methods in Modern Physics, Mathematische Nachrichten, Publicationes Mathematicae Debrecen, Monatshefte für Mathematik, Beirtäge zur Algebra und Geometrie.
Reviewer for AMS Mathematical Reviews.

PROFESSIONAL EXPERIENCE

2006-2007	Università degli Studi dell'Insubria Tutoring (16 hours) for the following courses: "Calcolo II", "Geometria I", "Algebra I"	Como, Italy
2011	Università degli Studi dell'Insubria Exercise classes (20 hours) for the course "Analisi Matematica II". Main lecturer: Prof. Alberto G. Setti	Como, Italy
2012	Università degli Studi dell'Insubria Exercise classes (10 hours) for the course "Geometria II". Main lecturer: Prof. Stefano Pigola	Como, Italy
2013	Università degli Studi dell'Insubria Exercise classes (16 hours) for the course "Geometria II". Main lecturer: Prof Stefano Pigola	Como, Italy
2015	Università degli Studi di Milano-Bicocca Exercise classes (24 hours) for the course "Geometria I". Main lecturer: Prof. Davide L. Ferrario	Milano, Italy
2017	Politecnico di Torino Exercise classes (20 hours) for the course "Algebra lineare e geometria". Main lecturer: Prof. C. Cumino	Torino, Italy
2017	Politecnico di Torino Exercise classes (20 hours) for the course "Algebra lineare e geometria". Main lecturer: Prof. M. Ferrarotti	Torino, Italy
2017	Politecnico di Torino Exercise classes (20 hours) for the course "Linear algebra and geometry". Language: English. Main lecturer: Prof. E. Carlini	Torino, Italy
2017	Politecnico di Torino Exercise classes (20 hours) for the course "Geometria differenziale e computazionale". Main lecturer: Prof. E. Musso	Torino, Italy

FUNDING INFORMATION

2014-2015	Participant to the PRIN Project 2010-2011: Varietà reali e complesse: geometria, topologia e analisi armonica Coordinator: Prof. Fulvio Ricci
2014	Participant to the INdAM-GNAMPA Project : Proprietà analitiche e spettrali di varietà pesate e applicazioni Coordinator: Debora Impera
2013	Coordinator of the INdAM-GNAMPA Project: Funzioni e Mappe armoniche: misura quantitativa dell'insieme critico, regolarità e problemi di Dirichlet Grant value: 2500,00 Eu
2010	Participant to the INdAM-GNAMPA Project: Teoria del potenziale non lineare su varietà Riemanniane e applicazioni geometriche Coordinator: Prof. Stefano Pigola

SOCIETY MEMBERSHIPS

Since 2010	Member of INdAM-GNAMPA

OTHER ACTIVITIES

Member of the organizing committee of the school:
Geometric Analysis on Riemannian and singular metric measure spaces
Como, Italy, July 11-15, 2016
Mini-courses by: H.-D. Cao, N. Gigli, T. Ilmanen, R. Schoen, C. Sormani, X.-P. Zhu.
Scientific Committee: G. Besson, S. Pigola, A. Setti, M. Troyanov.

Member of the organizing committee of the workshop:
A Geometry Day in Como 2015
Como, Italy, Jan 9, 2015
Talks by: Z. Djadli, A. Malchiodi, L. Mari, B. Nelli, R. Paoletti.

Member of the organizing committee of the workshop:
A Geometry Day in Como 2014
Como, Italy, Jan 10, 2014
Talks by: F. Bonsante, F. Coda-Marques, I. Holopainen, N. Gigli, C. Mantegazza, M. Rigoli.

Member of the organizing committee of the school:
Geometric Analysis on Riemannian and singular metric spaces
Como, Italy, Sept 30-Oct 5, 2013
Mini-courses by: S. Alexander, G. Carron, E. Hebey, U. Lang, A. Neves
Scientific Committee: G. Besson, S. Pigola, A. Setti, M. Troyanov.

CONFERENCES, WORKSHOPS AND SUMMER SCHOOLS

February 2017	Università di Torino, Torino, Italy *Differential Geometry Days*
February 2017	Scuola Normale Superiore, Pisa, Italy *Workshop su varietà reali e complesse:* *geometria, topologia e analisi armonica*
September 2016	Università di Cagliari, Cagliari, Italy *Complex and Riemannian Geometry Days*
July 2016	Lake Como School for Advanced Studies, Como, Italy *School in Geometric Analyisis* *Geometric Analysis on Riemannian and singular metric measure spaces*
April 2015	Universidad de Granada, Granada, Spain *Workshop on Geometric Flows*
March 2015	Goethe Universität, Frankkfurt am Main, Germany *Flow(ers) & Friends* *A workshop on Geometric Analysis*
January 2015	Università degli Studi dell'Insubria, Italy *A geometry day in Como*
November 2014	Università degli Studi di Milano, Milano, Italy *PDE's and Global Analysis @ UniMi*
October 2014	Levico Terme (TN), Italy *Progressi Recenti in Geometria Reale e Complessa - IX*
September 2014	Villasimius (CA), Italy *New trends in Differential Geometry 2014*
June 2014	Centro de Giorgi, Pisa, Italy *ERC School on Geometric Evolution Problems*
February 2014	Universidade Federal do Cearà, Fortaleza, Brasil *VII Workshop on Geometric Analysis*
January 2014	Università degli Studi dell'Insubria, Italy *A geometry day in Como*
October 2013	Institut Fourier, Grenoble, France *Autumn days in Grenoble* *Curvature in metric spaces*
October 2013	Lake Como School for Advanced Studies, Como, Italy *School in Geometric Analyisis* *Geometric Analysis on Riemannian and singular metric spaces*
June 2013	Universidad de Granada, Spain

	Variational problems and Geometric PDE's
Feb.-Mar. 2013	SCUOLA NORMALE SUPERIORE, PISA, ITALY *Workshop su varietà reali e complesse:* *geometria, topologia e analisi armonica*
January 2013	UNIVERSITÀ DEGLI STUDI DELL'INSUBRIA, ITALY *A geometry day in Como*
Sept.-Dec. 2012	INSTITUT HENRI POINCARÉ, PARIS, FRANCE *Research trimester in Conformal and Kälher geometry*
June 2012	ICTP, TRIESTE, ITALY *ICTP-ESF School and Conference on Geometric Analysis*
September 2011	UNIVERSIDAD DE GRANADA, SPAIN *VI International Meeting on Lorentzian Geometry*
May 2011	UNIVERSITÀ DI MILANO BICOCCA, ITALY *Geometria in Bicocca*
April 2011	CENTRO DE GIORGI, PISA, ITALY *Ricci Solitons Days in Pisa 2011*
February 2011	UNIVERSITÀ DI MILANO BICOCCA, ITALY 1st Bicocca HART *Harmonic Analysis and Related Topics*
February 2011	UNIVERSIDAD DE GRANADA, SPAIN *Spanish–Japanese Workshop on Differential Geometry*
September 2010	VERBANIA, ITALY Riemann International School of Mathematics *Nonlinear Differential Equations*
June–July 2010	CETRARO (CS), ITALY CIME course on *Ricci Flow and Geometric Applications*
May 2010	DRESDEN UNIVERSITY OF TECHNOLOGY, GERMANY The 8th AIMS Conference on *Dynamical Systems, Differential Equations and Applications*
September 2009	UNIVERSITÀ DI CAGLIARI, ITALY *A harmonic map fest*
June 2009	CENTRO DE GIORGI, PISA, ITALY Research Trimester on *Geometric flows and Geometric Operators*
April 2009	VERBANIA, ITALY Riemann International School of Mathematics *Advances in number theory and geometry*

RESEARCH ACTIVITY

So far, my research activity has mainly dealt with geometric PDE's on Riemannian manifolds. In particular, I've been focusing on the following topics:

- **Ricci solitons.**([PRiS], [PRRS], [MRR], [RV1], [RV2]) Let $(M, \langle\,,\,\rangle)$ be a Riemannian manifold. A Ricci soliton structure on M is the choice of a smooth vector field X (if any) satisfying the soliton equation $Ric + \frac{1}{2}\mathcal{L}_X \langle\,,\,\rangle = \lambda \langle\,,\,\rangle$. In the special case where $X = \nabla f$ for some smooth function $f : M \to \mathbb{R}$ we say that $(M, \langle\,,\,\rangle, \nabla f)$ is a gradient Ricci soliton with potential f. In this case the soliton equation reads $Ric_f = Ric + Hess(f) = \lambda \langle\,,\,\rangle$.

  Since the appearance of the seminal works of R. Hamilton, [18], and G. Perelman, [30], the study of gradient Ricci solitons has become the subject of a rapidly increasing investigation mainly directed towards two goals, classification and triviality.
  (i) In collaboration with Stefano Pigola and Alberto G. Setti, in [PRiS] we have studied triviality and rigidity results and curvature estimates for gradient Ricci solitons under weighted L^p conditions on the relevant quantities. These extend and generalize, often in a significant way, previous results (see e.g. [15, 31, 33, 42, 10]). Moreover, we show how techniques coming from stochastic analysis, such as stochastic completeness, in the form of the weak Omori–Yau maximum principle, parabolicity and L^p–Liouville type results, once translated into the setting of weighted geometry, are natural in the investigation of gradient Ricci solitons, and lead to elegant proofs of the above mentioned results.
  (ii) In collaboration with Stefano Pigola, Marco Rigoli, and Alberto G. Setti, in [PRRS] we have introduced an extension of the concept of gradient Ricci soliton, the Ricci almost soliton, allowing λ in the soliton equation to be a generic smooth function on the weighted manifold $M_f = (M, \langle\,,\,\rangle, e^{-f}d\mathrm{vol})$. In view of the fact that the soliton function λ is not necessarily constant, one expects that a certain flexibility on the almost soliton structure is allowed and, consequently, the existence of almost solitons is easier to prove than in the classical situation. This feeling is confirmed by a number of different examples of almost solitons. On the other hand we prove a rigidity result which indicates that almost solitons should reveal a reasonably broad generalization of the fruitful concept of classical soliton. In particular, one obtains that not every complete manifold supports an almost soliton structure. The investigation in [PRRS] is mainly concerned with triviality, pointwise curvature estimates and isolation phenomena for complete gradient Ricci almost solitons and the results follow from considering elliptic equations or inequalities for various geometric quantities on almost solitons and rely on analytical techniques. This is the same philosophy used e.g. by M. Eminenti, G. La Nave and C. Mantegazza in [15] or in [PRiS]. More specifically, one sees that the differential (in)equalities at hand naturally involve the f–Laplacian. Since the almost soliton equation means precisely that the f–Bakry–Émery Ricci curvature Ric_f is proportional to the metric tensor, we are naturally led to introduce a number of weighted manifolds tools whose range of application go beyond the investigation of almost solitons. An important istance of these tools is represented by the Omori–Yau maximum principle for the f–Laplacian under both weighted Ricci lower bounds and weighted volume growth conditions. Moreover we extended to almost solitons also some topological results known in the classical case ([39, 18, 27, 15]).
  (iii) While gradient Ricci solitons were widely studied, relatively little is known about generic Ricci solitons, that is, when the soliton field is not necessarily the gradient of a potential, and the majority of the results is concerned with the compact case. A first important difference is that, in the general case, we cannot make use of the weighted manifold structure which naturally arises when dealing with gradient solitons. The same applies for related concepts such as the Bakry–Émery Ricci tensor, whose boundedness from below with a suitable radial function, together with an additional assumption on the potential function gives rise to weighted volume estimates. These facts restrict the applicability of analytical tools such as the weak Omori–Yau maximum principle for the diffusion operator known as f–Laplacian, weighted L^p–Liouville–type theorems and *a priori* estimate. Nevertheless, in the general case the soliton structure is encoded in the geometry of an appropriate operator Δ_X that we shall call the X–Laplacian and that is defined on $u \in C^2(M)$ by $\Delta_X u = \Delta u - \langle X, \nabla u\rangle$.

Assuming a suitable growth condition on the vector field X, in collaboration with Paolo Mastrolia and Marco Rigoli, in [MRR] we obtained scalar curvature and volume estimates and a gap theorem for the traceless Ricci tensor. These results permit to deduce some interesting geometric consequences.
(iv) In collaboration with Giona Veronelli, in [RV1], we studied f–harmonic maps from non-compact manifolds into non-positively curved ones. Notably, we proved existence and vanishing results which generalize to the weighted setting part of Schoen and Yau's theory of harmonic maps, [34, 35]. As an application, we deduced information on the topology of manifolds with lower bounded Bakry–Émery Ricci tensor, and in particular of steady and expanding gradient Ricci solitons. Unlike most results so far known some topological theorem is obtained without requiring any assumption on the weight function f.
(v) In collaboration with Giona Veronelli, on [RV2], we established the existence of extremals for the Log Sobolev functional on complete non-compact manifolds with Ricci curvature bounded from below and strictly positive injectivity radius, under a condition near infinity. This generalizes a result previously obtained in [40] where it was assumed that the manifold had bounded geometry (i.e. the Riemann curvature tensor and all its covariant derivatives are bounded and that there is a uniform positive lower bound on the volume of geodesic balls of radius 1). When Ricci curvature is also bounded from above we get exponential decay at infinity of the extremals. As a consequence of these analytical results we establish, under the same assumptions, that non-trivial shrinking Ricci solitons support a gradient Ricci soliton structure. Our technique hence gives different conditions from those previously considered in [27] for the validity of a such a conclusion. On the way, we prove two results of independent interest: the existence of a distance-like function with uniformly controlled gradient and Hessian on complete non-compact manifolds with bounded Ricci curvature and strictly positive injectivity radius and a general growth estimate for the norm of the soliton vector field on manifolds with bounded Ricci curvature.

- **Quasi–Einstein manifolds.**([R1], [MR], [CMMR]) Another possible generalization of Einstein manifolds and gradient Ricci solitons, first considered by J. Case, Y.–S. Shu and G. Wei, [8], are quasi–Einstein manifolds. These are weighted manifolds M_f such that their modified Bakry–Émery Ricci tensor $Ric + Hess(f) - \frac{1}{k} df \otimes df$ is a constant multiple of the metric. The importance of these geometric structures comes from a problem (proposed in [2]) on the existence of Einstein manifolds realized as warped products. Indeed, in [26], D.–S. Kim and Y.–H. Kim proved a characterization of quasi–Einstein metrics as base metrics of Einstein warped product metrics.
(i) Exploiting this relationship, in [R1] and in [MR] (this latter written in collaboration with Paolo Mastrolia), we have obtained triviality and rigidity results for non–compact quasi–Einstein manifolds and Einstein warped products with non–compact bases and Einstein fibre. The proofs rely on the Omori–Yau maximum principle for the weighted Laplacian, a new gradient estimate for solutions of weighted Poisson–type equations and some Liouville–type theorems in the setting of weighted manifolds.
(ii) We have next faced the problem of the local characterization of locally conformally flat quasi–Einstein manifold. In particular, in collaboration with Giovanni Catino, Carlo Mantegazza, and Lorenzo Mazzieri, in [CMMR] we proved that any complete locally conformally flat quasi–Einstein manifold of dimension $m \geq 3$ is locally a warped product with $(m-1)$–dimensional fibers of constant curvature. Although the result was already known for gradient Ricci solitons (of which quasi–Einstein manifolds are a generalization), [41, 9, 6], the strategy of the proof is completely new and can be used as the main step to classify locally conformally flat shrinking and steady gradient Ricci solitons.

- **Self–shrinkers of the mean curvature flow.**([R2], [PR2]) By a self–shrinker of the MCF we mean a connected, isometrically immersed hypersurface $x : \Sigma^m \to \mathbb{R}^{m+1}$ whose mean curvature vector field $\mathbf{H}$ satisfies the equation $x^{\perp} = -\mathbf{H}$. Self–shrinkers play an important role in the study of mean curvature flow, since they describe all possible blow ups at a given singularity of a MCF. Let $f = \frac{|x|^2}{2}$ and denote by $\mathbf{A}$ the second fundamental form of the immersion. In [R2] we observed that from the self–shrinker equation one gets that

$Ric_f \geq 1 - |\mathbf{A}|^2$. This important estimate permits to apply, once again, the analytical tools developed so far in weighted geometry. Indeed the differential (in)equalities at hand on self–shrinkers involve the natural drifted Laplacian Δ_f on Σ_f.
(i) The classification of mean convex self–shrinkers began with [24], where G. Huisken showed that the only smooth closed mean convex self–shrinkers are round spheres. In the complete non–compact case G. Huisken, [23], also proved a classification theorem saying that the only possible smooth mean convex embedded self–shrinkers with bounded $|\mathbf{A}|$ and (extrinsic) polynomial volume growth are the cylindrical products $\mathcal{C}^{k,m-k}_{\sqrt{k}} = \mathbb{S}^k_{\sqrt{k}} \times \mathbb{R}^{m-k}$, $k = 0, \dots, m$. Recently T. Colding and W. Minicozzi, [13], showed that the hypothesis $|\mathbf{A}|$ bounded can be dropped. The hypothesis of polynomial volume growth in [13] is used to show that various weighted integral converge in order to justify various integration by parts and it is natural in the study of the singularities that a MCF goes through since any time–slice of a blow up of a closed MCF has polynomial growth. Nevertheless, looking only at the self–shrinker equation, it might be tought under what weaker conditions the conclusion in [13] still holds. In the main result in [R2] we replace polynomial volume growth with a weigthed L^2–condition on $|\mathbf{A}|$. The viewpoint of weigthed manifolds we adopt permits also to recover easily a classification result by H.D. Cao and H. Li, [6], where instead of the mean convexity condition it is considered a L^∞–type condition on $|\mathbf{A}|$.
(ii) It was conjectured bt H. D. Cao, [7], that every complete self–shrinker has extrinsic polynomial volume growth. By a result due to Q. Ding and Y. L. Xin, [14], later completed by X. Cheng and D. Zhou, [12], this is equivalent to the fact that the immersion is proper. Thus, by way of example, if Cao conjecture is true, then any complete self–shrinker in a ball of $\mathbb{R}^{m+1}$ should be compact. In this direction, in collaboration with Stefano Pigola, in [PR2] we moved some steps in getting information on the geometry of non–compact bounded self–shrinkers (if any) obtaining natural and general restrictions that force these hypersurfaces to be compact (e.g. global boundedness conditions, pinching conditions at infinity and $L^{p\geq m}$ conditions on $|\mathbf{A}|$). More generally we tried to understand the geometry of self–shrinkers which are confined in a connected region bounded by some dilated cylinder $\mathcal{C}^{k,m-k}_R$. This was done proving a general strong maximum principle for self–shrinkers. Actually, when $k = 0$, it is reasonable that a complete self–shrinker has a certain homogeneous distribution around $0 \in \mathbb{R}^{m+1}$ and, therefore, it should intersect every hyperplane through the origin. Moreover, by the maximum principle, the intersection should be transversal somewhere unless the self–shrinker is itself an hyperplane. For compact self–shrinkers these properties are easily verified and a similar conclusion can be obtained for complete self–shrinkers with a controlled extrinsic geometry. In [PR2] we pointed out natural geometric conditions that permit to recover the full conclusion of the compact case. Once we have understood that, to a certain extent, complete self–shrinkers intersect transversally a hyperplane through the origin, we were also able to deduce spectral information on the drifted Laplacian Δ_f whenever the intersection is compact, and some (extrinsic) volume growth condition is satisfied.

- **f–minimal hypersurfaces and translators of the MCF.** ([IR1], [IR2]) Following M. Gromov, [21], if we consider an isometrically immersed orientable hypersurface Σ^m in the weighted manifold M^{m+1}_f, we can define a generalization of the mean curvature vector field as $\mathbf{H}_f = \mathbf{H} + \left(\overline{\nabla} f\right)^\perp$. Since the weighted structure on M induces a weighted structure on Σ we can consider the variational problem for the weighted area functional. From variational formulae, one can see that Σ is f–minimal, namely a critical point of the weighted area functional, if and only if $\mathbf{H}_f$ vanishes identically. Clearly minimal hypersurfaces are a particular case of f–minimal hypersurfaces corresponding to the case $f \equiv const$. Moreover, self–shrinkers of the mean curvature flow are important examples of f–minimal hypersurfaces in the Euclidean space with the Gaussian density $e^{-\frac{|x|^2}{2}}$.
(i) The research on f–minimal hypersurfaces has just started and it has been already approached by many authors, see e.g. [17], [11], [16]. Much efforts have been devoted to the study of the stability properties. These are taken into account by spectral properties of the weighted Jacobi operator $L_f = -\Delta_f - \left(|\mathbf{A}|^2 + \overline{\mathrm{Ric}}_f(\nu,\nu)\right)$. The most up to date result, proved by X. Cheng, T. Mejia, and D. Zhou, [11], states that there exist no L_f–stable com-

plete f–minimal hypersurfaces Σ immersed in a complete weighted Riemannian manifold M_f with $\overline{\mathrm{Ric}}_f \geq k > 0$, provided $\mathrm{vol}_f(\Sigma) < +\infty$. In collaboration with Debora Impera, in [IR1], we generalized this result considering growth conditions on the intrinsic weighted volume of geodesic balls. Furthermore, in the instability case, exploiting the oscillatory behaviour of solutions of some ODEs that naturally arise in this setting, we investigate general geometric restrictions for the finiteness of the weighted index of the f–minimal hypersurface, that is, the maximum dimension of the linear space of compactly supported deformations that decrease the weighted area up to second order. Moreover, exploiting a weighted version of a finiteness result and the adaptation to this setting of Li–Tam theory, we investigated the topology at infinity of f–minimal hypersurfaces. On the way, we prove a new comparison result in weigthed geometry and we provided a general weighted L^1–Sobolev inequality for hypersurfaces in Cartan–Hadamard weighted manifolds, satisfying suitable restrictions on the weight function and on its radial derivative.
(ii) By a translator of the MCF we mean a connected isometrically immersed complete hypersurface $x : \Sigma^m \to \mathbb{R}^{m+1}$ whose mean curvature vector field $\mathbf{H}$ satisfies the equation $\mathbf{H} = v^\perp$ for some fixed unit length vector $v \in \mathbb{R}^{m+1}$. These hypersurfaces correspond to translating solitons of the MCF, and play a key role in the study of slowly forming singularities. Translators of the MCF turn out to be f–minimal hypersurfaces in the Euclidean space $\mathbb{R}^{m+1}$ with the density $e^{\langle v,x\rangle}$. In collaboration with Debora Impera, in [IR2], we highlighted how the realm of weighted manifolds and f–minimal hypersurfaces can naturally give strong enough characterization and topological results for translators. About the firsts, we proved a rigidity theorem for f–stable translators under a weighted L^2–condition on the norm of the second fundamental form. This result, in particular, permits to strengthen previous rigidity results, see [36]. Concerning the latters, exploiting in an essential way the correspondence, discovered by K. Smoczyk, [37], between translators of MCF in $\mathbb{R}^{m+1}$ and minimal hypersurfaces in the manifold $\mathbb{R}^{m+1} \times \mathbb{R}$, endowed with a suitable warped product metric, we obtain the validity of a weighted $(m+1)$–dimensional L^1 Sobolev inequality on translators. The validity of this inequality permits to obtain very neat results on the topology at infinity of f–stable translators and translators with finite f–index. The main tools here are again weighted versions of the Li-Tam theory and of an abstract finiteness results, which were developed in [IR1]. Moreover, under the geometric assumption that the translator is contained in a halfspace determined by an hyperplane orthogonal to the translating direction v, we were able to guarantee the validity of the standard (m–dimensional) L^1–Sobolev inequality. This permit to get topological conclusion on translators also in case, instead of assuming the finiteness of the f–index, we are asking that the hypersurface has finite weighted total curvature.

- **Myers' type theorems.** ([MRV]) We analyzed the roots of the solutions of the special ODE $u''(t) + k(t)u(t) = 0$. The existence of (at least) a zero for a solution of this differential equation implies the compactness of the m–dimensional manifold M, once $(m-1)k(t) = \mathrm{Ric}_\gamma(t)$, where Ric_γ is the radial Ricci curvature of M. Developing an idea of E. Calabi, [5], in collaboration with Paolo Mastrolia and Giona Veronelli, in [MRV] we gave a new compactness result for complete manifolds with Ricci curvature lower bounded by a negative constant, thus extending previous results in [1, 19, 28]. Similar techniques permit to analyze also the zeros of the more general problem $(v(t)u'(t))' + k(t)v(t)u(t) = 0$, for some "weight" function $v(t)$. Even in this case, we studied assumptions which guarantee existence of a zero and oscillation of the solutions. As previously observed by B. Bianchini, L. Mari and M. Rigoli, [3], an application of these results yields new estimates on the bottom of the spectrum and on the index of some Schrödinger operator on Riemannian manifolds.

- **Evolution by Ricci flow of geometric tensors.** ([MMR]) Complete gradient steady Ricci solitons play an important role in the study of Hamilton's Ricci flow as they correspond to translating solutions, and often arise as type II singularity models. Thus one is interested in classifying them and understanding their geometry. R. Bryant has discovered a steady Ricci soliton in dimension 3, which is rotationally symmetric and has positive curvature operator. Bryant's construction can be adapted to higher dimensions. Inspired by the uniqueness property in dimension 2 of the cigar soliton, in [31], Perelman conjectured that the Bryant soliton is (up to scaling) the only complete non–compact three–dimensional gradient steady Ricci soliton with positive sectional curvature which satisfies a non–collapsing assumption at infinity (namely $\mathcal{K}$–noncollapseness). In [6]. H.D. Cao and Q. Chen proved uniqueness in dimension $m \geq 3$ under the additional assumption that (M^m, g) is locally conformally flat. The same result was proved indipendently by G. Catino and C. Mantegazza, [9], in dimension $m \geq 4$. Then, increasingly weaker conditions were considered, until, very recently, S. Brendle gave a proof of the conjecture without any further assumption, [4].

The proof strategy adopted in [9] to obtain uniqueness of locally conformally flat steady gradient non–flat Ricci solitons (for $m \geq 4$) is based on the computation of the evolution of the Weyl tensor under the Ricci flow. Inspired by this technique, in collaboration with C. Mantegazza and S. Mongodi, [CMM], we have computed the evolution of the Cotton and the Bach tensor under the Ricci flow of a Riemannian manifold, with particular attention to the three–dimensional case and we discussed some applications. Beyond their own interest, these computations suggest an alternative way to address Perelman's conjecture.

- **Obata's type characterization theorems.**([PR1]) We studied rigidity properties of Riemannian manifolds supporting solutions of particular second order differential systems. In particular, in collaboration with Stefano Pigola, in [PR1] we have considered more general differential systems than the classical ones of Obata's type ([29, 25, 38]), allowing the coefficient for the linear term to be a function (non–necessarily constant) of the distance function from a reference point. This permitted us to extend the characterization of space–forms to the case of model manifolds. We generalized also some metric rigidity results ([20]) by means of equations involving (gradient) vector fields.

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